INTERDISCIPLINARY MATHEMATICAL MODELLING MEETS CIVIC EDUCATION

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CONSORTIUM

Leibniz University Hannover

Welfengarten 1

30167 Hannover, Germany

Bastian Vajen

b.vajen@ipw.uni-hannover.de

Babeș-Bolyai University

Strada Mihail Kogălniceanu 1

Cluj-Napoca 400000, Romania

Florin Fesnic

fesnic@fspac.ro

Norwegian University of Science and Technology

Høgskoleringen 1

7491 Trondheim, Norway

Heidi Strømskag

heidi.stromskag@ntnu.no

Paderborn University

Warburger Str. 100

33098 Paderborn, Germany

Lara Gildehaus

lara.gildehaus@math.uni-paderborn.de

University of Vienna

Universitätsring 1

1010 Wien, Austria

Marco Mogiani

marco.mogiani@univie.ac.at

CONTENT

Authors: Lara Gildehaus; Michael Liebendörfer; Heidi Strømskag; Bastian Vajen, Frode Rønning; Yael Fleischmann; Alessandra Santoianni; Clara C. Berger; Florin Fesnic

1. Introduction: Our aim

The goal of the CiviMatics project is to offer educational tools to enhance the civic competences of European citizens, especially when it comes to interacting with global issues facing European societies. Our focus is on developing tools to enable citizens to better understand complex societal issues, such as climate change, by integrating different perspectives on the issue from scientific fields to create learning environments that foster a better understanding of the issue at hand. We are basing this approach on the principle of civic education that sets the mature citizen as the goal of educational processes. Maturity, or *Mündigkeit*, is understood as political, moral, and ethical autonomy (Henkenborg, 2012, p. 28). This includes the "freedom to make public use of one's reason in all matters" as a condition for the emergence of maturity – as well as the capacity and courage "to use one's own understanding without the guidance of another" (O'Neil, 2002, p. 251; see also Kant, 1781). Climate change is only one example of a variety of societal issues that are not exclusively influenced by rational discourse based on scientific insights, but also by deliberate misinformation and conspiracy theories (Douglas & Sutton, 2015). To combat such issues, it is essential to facilitate the acquisition of competences that enable individuals to properly analyse the issues at hand, judge them based on reason and by taking different perspectives into account and finally to act independently and in accordance with their own judgement. Such competences are aimed at helping learners to participate in a democratic political system (Reinhardt, 2013, p. 99).

2. The need for an integration of mathematics education and civic education

Political decisions addressing multifaceted issues are becoming increasingly complex and have to take into account a large number of different needs. Applying mathematics not only as pure science, but also to guide one's understanding of natural and social phenomena and using mathematics to aid in decision making processes accordingly has become more widespread in our complex information society and with it, the need for citizens' understanding of it for an informed and productive participation (Ojose, 2011, p. 99). In this context, "the purpose of mathematics education should be to enable students to realise, understand, judge, utilise and also

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perform the application of mathematics in society" (Niss, 1983, p. 248). This includes the development and application of models for political decision making, but also the reflection on the principles and assumptions influencing the transformation of societal issues into mathematical models.

One issue that highlights the use of mathematics for understanding complex phenomena, as well as the need to comprehend its application for making informed decisions, is the issue of climate change. A concise definition of climate change is given by the *Intergovernmental Panel on Climate Change* (IPCC):

Climate change in IPCC usage refers to a change in the state of the climate that can be identified (e.g. using statistical tests) by changes in the mean and/or the variability of its properties, and that persists for an extended period, typically decades or longer. It refers to any change in climate over time, whether due to natural variability or as a result of human activity. (IPCC, 2008, 30).

According to Barwell (2013), understanding a complex phenomenon such as climate change involves mathematics for describing, understanding, predicting and communicating the problems at hand. However, as one of the most striking problems that peoples and governments across the world are facing, it is an issue that not only incorporates the field of mathematics, but also the political sphere. It is an issue that has the potential to change the social and political order of the world (Beck, 2014, p. 75). Thus, it is not surprising that an education for sustainability has become a focal point at various educational levels. Its goal is to help citizens to understand the issues societies are facing, to empower them to advance specific changes toward sustainable development as well as encourage the citizens to critically question the status quo and enable them to investigate unanswered, but pressing, questions (Michelsen & Overwien, 2020, p. 562). However, enabling citizens to understand such a multifaceted issue like climate change relies on diverse approaches to teaching that incorporate the various aspects of the issue. To understand the science behind it, the predictions being made and their validity, the societal issues being created as well as the political discourse and decisions that will be connected to it, requires an interdisciplinary approach to teaching and learning. It is thus our goal to create didactic approaches that combine different disciplines – mathematics and political science – and their perspectives on a complex issue such as climate change. Both principles of civic education, for example the principles of problem orientation, and mathematics education, with theories such as the ATD and its study and research paths or the modelling cycle, offer opportunities for interdisciplinary learning processes, which we will detail below.

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3. Central concepts

3.1 Model and Modelling

The notion of a model relies on the notion of *system*, that is, a reality subject to its own laws. A model is the result of a transformation of a system, usually a simplification, which is supposed to help in generating knowledge about the studied system. In practice, to answer a question relating to a system, one tries to build up a model which is easier, safer, and quicker to study than the system itself. Models are thus used for answering questions or exploring facts, guided by research questions. Models always have a descriptive function, but they can also contain statements about what an individual should do. If this is the case, models are referred to as prescriptive, or normative. Depending on the case, a model can thus have descriptive, normative or prescriptive *uses*. For example, prognostic models (e.g., models of greenhouse gas emissions) are classified as descriptive, but they can be applied in a normative way.

Modelling is understood as the very process of building a model of a system and using it to answer questions about the system at stake. There are different tools for modelling, such as the modelling cycle (Blum & Leiß, 2005) and the Herbartian schema in the framework of the ATD (Chevallard, 2020). Insofar as modelling involves valuations, we speak of normative modelling. Models can be prescriptive but non-normative (e.g., a cake recipe), but very often prescriptive models will be normative, because certain actions or outcomes are treated as desirable. It is important to recognize that descriptive models can also be normative if, for example, the descriptive categories used are judgmental ("normal weight") or implicitly value certain actions or outcomes. For example, a model that relates CO2 emissions in a country to people suggests different consequences than a model that relates CO2 emissions to economic output. In shorter terms: While a model can be used or modelled in a purely descriptive way, i.e. as model *of* something, as is often the case in physics, for example, it can also be used or modelled in a normative way, being a model *for* something (Hilgers, 2010).

3.2 Principles of civic education

Even though models are used on a regular basis in civic education, didactic discourses about the structure of models as well as their use for educational processes are lacking. When using models in civic education, the goal is mainly to exemplify political processes and help students to understand their principle and structure. One prevalent model used to exemplify the process of political decision making is the *policy cycle*. This model describes the policy process as evolving through a sequence of distinct stages. Initially introduced as a normative model in political science aiming to provide an ideal framework for planning and decision making, it has developed into a widely applied framework to organize research on public policy (Jann & Wegrich, 2007). It was also introduced into civic education to help teachers and students grasp real political situations in their complexity, interdependence, and formative elements. The phases of the political cycle, and the categories that influence it, are reformulated into key questions guiding the process of understanding political decision making (Massing, 1995, pp. 86). Such categories are for example the division of the political domain into polity (form), policy (process) and politics (content) (Oberle, 2017, p. 25). The policy cycle can be seen as a tool to exemplify the processual structure of political decision-making, which may be influenced by a variety of different variables, but consists of a distinct pattern (Massing, 1995, p. 88). Thus, the use of models such as the policy cycle is twofold: On the one hand, the models should provide students which a simplified, and thus somewhat flawed, picture of reality. On the other hand, they can be used as a tool for analysis, by comparing real political processes with elements of the model and offering a basis for inquiry.

Connected to the use of models such as the policy cycle are often other principles of civic education, such as the problem orientation (Ackermann et al., 2018, pp. 31–33). Problem orientation states that (political) problems and their treatment are, as objects of politics, at the same time objects of civic education (Goll, 2014, p. 258). By taking up political problems (e.g., climate change) and making them the focus of learning about politics, decisions are made about the methodological form of the teaching-learning process. The teaching of problems aims at problem-solving thinking and, if successful, promotes a high degree of judgment competence and political maturity in the learners (ibid). To achieve this, Goll (2014) proposes that most approaches have three methodical steps: The analysis of the situation, the discussion of possibilities and the formation of a judgement (p. 263). Such principles of civic education are aimed at helping students acquire the ability to form a political judgement, which represents the core of political education processes (Juchler, 2012, p. 24). The importance of independent political judgment arises from the close connection between the ability to judge and the concept of political maturity. The goal of civic education is to contribute to the development of "political, moral, and ethical autonomy" through political maturity, which as part of self-determination always requires the ability to make independent judgments (Henkenborg, 2012, pp. 28–29). In this context, learners should be enabled to make independent assessments of political, economic, or social issues while weighing different criteria (Reinhardt, 2018, p. 24).

However, the ability to judge cannot be regarded as a stand-alone competence, but has to be integrated into the subject-didactic "triad" of political analysis competence, judgment competence, and action competence (Henkenborg, 2012, pp. 32–34). On the one hand, judgment is therefore dependent on a well-developed analytical competence, since a well-founded judgment appears impossible without penetrating social and political facts and structures; on the other hand, it is also linked to the competence to act, since the rational and independent judgment represents the basis of the political action of democratic citizens. Using societal problems as a topic for civic education processes can help learners understand the causes of such issues and enable them to analyse the political processes that are involved in solving them. Models can be a useful tool to facilitate learning in this context, both regarding the analysis of an issue as well as the political steps that can be taken to solve it. For an issue such as climate change, the understanding of which is dependent on mathematical and political competences, it can be beneficial to combine principles of civic education and mathematics education to enable learners to grasp the issue, analyse possible solutions and act in accordance with their own judgment.

4. POSSIBILITIES FOR INTERDISCIPLINARY LEARNING

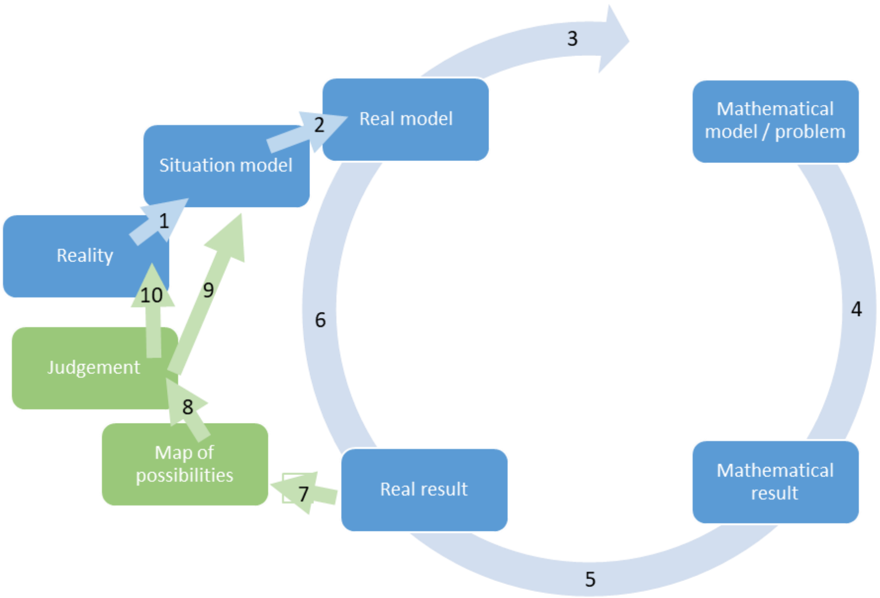
In this section, we present two approaches to interdisciplinary learning: first, a normative modelling cycle combining civic education and the modelling cycle; and, second, Study and Research Paths utilizing the Herbartian schema.

4.1 A NORMATIVE MODELLING CYCLE

The combination of civic education and mathematics requires adaptations of established modelling cycles in order to make political analysis and judgment explicitly visible. Previous representations aim at a mathematical result, which is often checked for its correctness at the end (e.g. by a measurement or an experiment). This unambiguity and verifiability is not given in normative modelling. Our proposal on the methodological level therefore consists of a combination of the steps of political didactic problem orientation and mathematical modelling, which are made visible in a common model. The basis for this is a modelling cycle that already contains a so-called situation model, i.e. a mental representation of the situation (Borromeo Ferri, 2006, p. 92; Blum & Leiß, 2005). Normative modelling, however, requires more, namely a political analysis of the situation (in addition to a mathematical analysis), a discussion of political possibilities, and a judgment formation as subsequent steps. At a minimum, the discussion of political possibilities requires that different possibilities emerge from the mathematical models or that they can be considered from the very beginning. Therefore, the question of selecting models or families of models arises. Policy analysis also requires identification of the interests of involved stakeholders. Neither alternative models nor affected interests emerge on their own. To incorporate these steps, we suggest a new modelling cycle, based on established approaches but adding additional steps for the modelling processes (see Figure 1).

The first step, constructing (1), does not involve conscious steps, but suggests that in normative modelling we may need to consider different conceptions of reality if we are to negotiate solutions socially. The following step, simplifying (2) is probably the most significant one, since the question which parts of the situation model are included at all and how interrelationships are simplified essentially determine the result. Here, alternatives have to be considered, their consequences for the model have to be estimated and they have to be classified with regard to political interests. The following mathematisation (3) is in itself a technical, unambiguous step, provided that the real model is specified precisely enough. In practice, however, the real model is specified more concretely in this step, so that simplifications similar to those in (2) are to be expected here as well. The mathematical work (4) will rarely provide starting points for the political discussion. Although alternative actions exist here (e.g., obtaining solutions algebraically or numerically), the differences, if any, should be irrelevant. Interpreting (5) should also be more of a technical step because it initially involves only the translation of mathematical variables, functions, etc. into reality.

However, generalizations could be made at this step, concerning e.g. model assumptions or restrictions of variable ranges. Moreover, the presentation of the results will very often suggest actions, at least implicitly. Such (normative) statements can never be the result of a mathematical calculation and are therefore made explicit in our model through further steps. First, the different possibilities and the different implications in terms of stakeholder interests should be noted by reflecting and critiquing the modelling just described (7). This will lead to a "map of possibilities". Next, all learners should form their own judgments by weighing interests (8). Finally, we should acknowledge that our decision might have an impact on the world as we assume it to be at that moment (in terms of our situation model; 9) and as it is (reality; 10).



1. Constructing/ analyzing/understanding
2. Simplifying/ structuring
3. Mathematizing
4. Working mathematically
5. Interpreting
6. Considering alternatives
7. Political analysis
8. Judgement formation
9. Assumed effect
10. Effect

Figure 1 – An interdisciplinary modelling cycle

The presented cycle can be helpful for the creation of models as well as for the analysis of modelling, because it explicitly points out working steps. For example, arguments that one has "recalculated" certain effects are made discussable. Different real results are usually not based on different interpretations (5) or mathematical solutions (4), but partly on mathematisations (3) and especially simplifications (2) as well as perceptions of reality (1), which all have to be discussed explicitly. In the process, it may be possible to identify the interests of the actors concerned, which frame such assumptions.

4.2 Study and Research Paths (SRPs) and the Anthropological Theory of the Didactic (ATD)

The ATD postulates that any activity related to the production, diffusion, or acquisition of knowledge should be interpreted as an ordinary human activity, and thus proposes a general model of human activities built on the notion of *praxeology*. This is a key notion in the ATD, explained like this:

> *A praxeology is, in some way, the basic unit into which one can analyse human action at large. […] What exactly is a praxeology? We can rely on etymology to guide us here – one can analyse any human doing into two main, interrelated components: praxis, i.e. the practical part, on the one hand, and logos, on the other hand. "Logos" is a Greek word which, from pre-Socratic times, has been used steadily to refer to human thinking and reasoning – particularly about the cosmos. […] One fundamental principle of the ATD [states that] no human action can exist without being, at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such an explanation or justification may be cast. […] Of course, a praxeology may be a bad one, with its "praxis" part being made of an inefficient technique – "technique" is here the official word for a "way of doing" – and its "logos" component consisting almost entirely of sheer nonsense – at least from the praxeologist's point of view! (Chevallard, 2006, p. 23).*

A praxeology in the ATD is a unit composed of four components (Chevallard, 2019): T, τ, θ and Θ (sometimes referred to as «the four t-s»). T (Latin capital letter t) is a *type* of tasks, τ (Greek tau) is a technique (or a set of techniques) to solve the tasks, θ (Greek theta) is a *technology* (i.e. a discourse) to describe and explain each technique, and Θ (Greek capital theta) is a theory that justifies the technology. T and τ belong to the praxis block of a praxeology, whereas θ and Θ belong to the logos block. A praxeology $\wp$ is thus written: $\wp = [T / \tau / \theta / \Theta]$. *A priori* praxeological analyses will be important for classroom experiments in the proposed project, where praxeological models of the knowledge at stake will be instrumental in designing interventions to be implemented in the classroom. Praxeological analyses can also be done *a posteriori* to analyse the building up of a praxeology during the solution of a problem (e.g. Strømskag, in press).

4.2.1 Methodology and didactic paradigm

The conceptualization developed in the ATD for questioning the world essentially boils down, from a methodological viewpoint, to the so-called *Study and Research Paths, SRPs*, which can be regarded as falling under the label of *didactic engineering* (Barquero & Bosch, 2015). In an SRP, there is a dialectic between "research" and "study" which characterizes any learning activity. "Research" refers to inquiry or problem solving while "study" designates the consultation of existing (and available) knowledge as initiated not only by the teacher but also by the students. The term "path" designates the possibly open trajectories followed in an experimentation of the SRP.

In our interdisciplinary approach, we will use the design of SRPs as a new *didactic tool* to teach mathematical modelling with a double purpose: to make students aware of the rationale of the mathematical content they have to learn and to connect this content through the study of open

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modelling questions. This is motivated by a desire to take steps towards a new didactic paradigm, that of *"questioning the world"* (Chevallard, 2015). The paradigm of visiting works underpins general education (as against vocational education, it seems). It involves a transmission style of teaching and amounts to "visiting" works that will generally never be met again. Forgetting these works is intrinsic to it, for their study is primarily valued as allegedly "formative of the mind". The paradigm of questioning the world involves a break with the paradigm of visiting works. However, when you inquire you also have to study works, and in some cases, it resembles the visiting of works. But in the context of an inquiry, it is the criterion of being *relevant* that matters (not the importance of this work as defined by the curriculum) – the relevance being defined by the capacity of this work to *provide answers* (or parts of answers) to questions raised by the inquiry.

4.2.2 Study and research paths

In an SRP, students collaborate in teams to answer *generating questions* given to them by the teacher who is the supervisor of the inquiries. An SRP with a generating question Q will involve students' investigations through activities like these:

- using (tentative, partial) answers found in the written literature (textbooks, articles, etc.)
- using Internet sources and digital resources (videos, etc.)
- studying new questions Q_j generated by the study of Q and by the components mentioned above
- collecting sets of data through diverse types of research
- studying all kinds of "works" other than those already mentioned (theories, experimental schemes, historical studies, reports, etc.)
- writing and presenting (to other teams and the supervisor) preliminary reports, accounting for the progress made by the team in answering Q
- giving feedback to other teams on their preliminary reports
- writing a final report accounting for the *answer* A to the question Q and the *tools* used to arrive at A

The pedagogic organization of an SRP will be as follows: The generating question Q of an SRP is presented by the teacher in charge of an *SRP workshop* to students who work in teams of two or three students. The SRP workshop regularly takes the form of a *seminar* in which student teams present the progress of their work on the question Q that was assigned to them, and present derived questions Q_i. It is in this seminar that the synthesis emerges specifying the expected answer *A of the class* to the question *Q*. The teacher in charge of the class considers the derived questions collected from the teams and reviews the progress of the teams' work. He/she makes decisions on any possible intervention on his/her part with the teams or the class (within the framework of the class's seminar).

The new paradigm is based on three principles related to curricula (Chevallard, 2018): First, every human community has duties towards its members. An essential duty is that of defining and implementing a community curriculum to ensure that all members of the community are *enabled to think and act appropriately*, in a way beneficial to themselves and to others, in the different social worlds (in particular the worlds of family, profession, and citizenship) in which they are or will be led to live. This can be connected to Lange's (2008) competence area of social learning and Prints competences for a democratic citizenship (2013). These involve among others the belief in social justice and the equality and equal treatment of citizens and the skills to build coalitions as well as to cooperate and be able to live and work in a multicultural environment (Print, 2013, p. 44-46). Second, the community's curriculum should aim to enable the members of the community, individually or in the form of collectives, to *identify, formulate and respond to the questions they face*. This is connected to democratic competences such as the capability to critically examine information and to be able to evaluate a position or decision, take a position, and defend a position (Print, 2013, p. 45). Third, to achieve this goal, the community shall define (and revise regularly) a curriculum *core made up of questions* that members of the community "have the right not to be allowed to avoid" (Gagnon, 1995). The aim of this is to create an understanding for differences in societies and thereby promote interaction and communication, violence prevention, cooperation, and conflict resolution, as well as to the recognition and acceptance that others might see things differently.

4.2.3 Study and research paths

A praxeological function that inquiring into a question *Q* assumes is to lead to studying all sorts of works (including derived questions Q_i). How can we describe what happens in a didactic system *S* when a class *X* studies a question *Q* under the supervision of teacher(s) *Y* ? The model provided by the ATD is the *reduced Herbartian schema*: $S(X; Y; Q) \hookrightarrow A$ (Chevallard, 2020). Here, *A* is the answer to the question *Q* that the didactic system is expected to produce. The answer *A* is usually written with a heart ♥ in superscript: $S(X; Y; Q) \hookrightarrow A^{\heartsuit}$ – this hints at the fact that this answer will be "at the heart" of the didactic system, the "authorised" answer to question Q (at least for some time).

Chevallard (2020) explains that the next step in building up a model of what happens in the didactic system studying the question Q, is the introduction of the didactic *milieu*, M, which is the set of material and immaterial tools that the class gathers in order to carry out their inquiry into question Q. The reduced Herbartian schema then becomes the *semi-developed* Herbartian schema: $[S(X; Y; Q) \curvearrowright M] \hookrightarrow A^{\heartsuit}$. Here, the didactic system is seen to create the milieu M and to produce the answer $A^{\heartsuit}$ by drawing upon the milieu M. In the quest for an answer A to the question Q, three main components stick out. The first is the search – "in the literature" and, in particular, on the Internet – for existing answers offered by other persons or institutions. Such answers are usually denoted by $A^{\lozenge}$, which can be read "A diamond" – the rhombus denoting the "hallmark" of some institution or person. A teacher (in direct instruction) or a textbook or a webpage are thus institutions that, de facto, "hallmark" their answers to the questions they tackle. At his stage, the milieu M is therefore to be written thus: $M = \{A_1^{\lozenge}, A_2^{\lozenge}, \ldots, A_m^{\lozenge}, \ldots\}$.

To draw upon the answers $A_i^{\lozenge}$ $(1 \leq i \leq m)$, the didactic system has recourse to works of various kinds, like theories, experiments, essays, etc. (Chevallard, 2020). Therefore the milieu is now to be written: $M = \{A_1^{\lozenge}, A_2^{\lozenge}, \ldots, A_m^{\lozenge}, W_{m+1}, W_{m+2}, \ldots, W_n, \ldots\}$. To use these works, the student needs to study them. What does it mean to study a work W (which is not itself a question) in order to study question Q? Such a study consists in studying a number of *questions* Q_w about the work under study. Thus, the study of any work boils down to the study of *questions*. The set of questions Q_w depends on the inquiry being conducted: as a general rule, they may differ according to the generating question Q and the way the inquiry into it proceeds. Much more generally, every single item in the milieu is bound to raise questions that, up to a point, the didactic system will have to study. Then the milieu M takes on the following appearance: $M = \{A_1^{\lozenge}, A_2^{\lozenge}, \ldots, A_m^{\lozenge}, W_{m+1}, W_{m+2}, \ldots, W_n, Q_{n+1}, Q_{n+2}, \ldots, Q_p\}$.

As explained above, the Herbartian schema is a (dynamic) representation of an inquiry. We will use it in the CiviMatics project as a tool to regulate and analyse inquiries. For our focus on normative models, this means that co-disciplinary inquiries will be conducted in the form of "normative (or prescriptive)" uses of mathematical modelling. This means that assumptions and limitations originating in political/civic as well as mathematical constraints will be explicated and taken into account when conducting the modelling. An important aspect is the productivity of the model – the fact that it produces *new knowledge* about the system being modelled – and this requires a certain ‘fit’ or ‘adaptation’ to the system.

5. Summary and Outlook

The ability to make autonomous political judgments, which is a central component of political maturity and thus also of political education, is dependent on extensive analytical competence, because without an understanding of social and political issues, a well-founded judgment seems impossible. Complex issues and their solutions, like climate change, require an understanding of mathematics and civics to properly analyse them and form an informed judgment. Mathematics education and civic education didactics offer a high potential for cooperation in the teaching of complex societal problems due to similar objectives and the reference to comparable concepts, such as the concept of maturity. The presented models are methodological proposals for the connection of political problem orientation and mathematical modelling, which offer potentials for both disciplines. The realization of the opportunities of such an approach is dependent on the development of appropriate didactic materials and practical testing in the classroom, which will also be developed during the CiviMatics project.

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6. LITERATURE

Ackermann, P., Breit, G., Cremer, W., Massing, P., & Weinbrenner, P. (2018). *Politikdidaktik kurzgefasst: 13 Planungsfragen für den Politikunterricht*. *Wochenschau Politik*. Schwalbach/Ts.: Wochenschau-Verl.

Barquero, B., & Bosch, M. (2015). Didactic engineering as a research methodology: From fundamental situations to study and research paths. In A. Watson & M. Ohtani (Eds.), *New ICMI Study Series. Task Design In Mathematics Education: An ICMI study 22* (pp. 249–272). Dordrecht, Netherlands: Springer International Publishing.

Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, *15*(1), 1–16.

Beck, U. (2015). Emancipatory catastrophism: What does it mean to climate change and risk society? *Current Sociology*, *61*(1), 75–88.

Blum, W., & Leiß, D. (2006). Filling up – In the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In M. Bosch (Ed.), *Proceedings of the Fourth Conference of the European Society for Research in Mathematics Education* (pp. 1623–1633). Barcelona, Spain: Universitat Ramon Llull and ERME.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM – Mathematics Education*, *38*, 86–95.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the Fourth Conference of the European Society for Research in Mathematics Education* (pp. 22–30). Barcelona, Spain: Universitat Ramon Llull and ERME.

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges* (pp. 173–187). Berlin, Germany: Springer International Publishing.

Chevallard, Y. (2018). The curricular conundrum in the light of the ATD. In Y. Shimizu & R. Vithal (Eds.), *The Twenty-fourth ICMI Study: School Mathematics Curriculum Reforms: Challenges, Changes and Opportunitie* (pp. 213–220). Tsukuba, Japan: University of Tsukuba.

Chevallard, Y. (2019). Introducing the Anthropological Theory of the Didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, *12*, 71–114.

Chevallard, Y. (2020). Some sensitive issues in the use and development of the anthropological theory of the didactic. *Educação Matemática Pesquisa*, *22*(4), 13–53.

Douglas, K. M., & Sutton, R. M. (2015). Climate change: Why the conspiracy theories are dangerous. *Bulletin of the Atomic Scientists*, *71*(2), 98–106.

Gagnon, P. (1995, December). What Should Children Learn? *The Alantic*. https://www.theatlantic.com/magazine/archive/1995/12/what-should-children-learn/376491/

Goll, T. (2014). Problemorientierung. In W. Sander (Ed.), *Reihe Politik und Bildung: Band 69. Handbuch politische Bildung* (4th ed., pp. 258–266). Schwalbach/Ts.: Wochenschau Verlag.

Henkenborg, P. (2012). Politische Urteilsfähigkeit als politische Kompetenz in der Demokratie: der Dreiklang von Analysieren, Urteilen Handeln. *Zeitschrift Für Didaktik Der Gesellschaftswissenschaften*, *3*(2), 28-50.

Jann, W., & Wegrich, K. (2007). Theories of the Policy Cycle. In F. Fischer & G. Miller (Eds.), *Public Administration and Public Policy. Handbook of Public Policy Analysis: Theory, Politics, and Methods* (pp. 43–62). Routledge.

Juchler, I. (2012). Politisches Urteilen. *Zeitschrift Für Didaktik Der Gesellschaftswissenschaften*, *3*(2), 10–27.

Kant, I. (2015). *Die drei Kritiken: Kritik der reinen Vernunft (1781/87) ; Kritik der praktischen Vernunft (1788) ; Kritik der Urteilskraft (1790)*. Köln: Anaconda.

Lange, D. (2008). Citizenship Education in Germany. In V. B. Georgi (Ed.), *Schriftenreihe / Bundeszentrale für Politische Bildung: Vol. 666. The making of citizens in Europe: New perspectives on citizenship education* (89–95). Bonn: Bundeszentrale für Politische Bildung.

Massing, P. (1995). Wege zum Politischen. In P. Massing & G. Weißeno (Eds.), *Schriften zur politischen Didaktik: Vol. 24. Politik als Kern der politischen Bildung: Wege zur Überwindung unpolitischen Politikunterrichts* (pp. 61–98). Wiesbaden: VS Verlag für Sozialwissenschaften.

Michelsen, G., & Overwien, B. (2020). Nachhaltige Entwicklung und Bildung. In P. Bollweg, J. Buchna, T. Coelen, & H.-U. Otto (Eds.), *Handbuch Ganztagsbildung* (557-574). Wiesbaden: Springer Fachmedien Wiesbaden.

Niss, M. (1983). Considerations and experiences concerning integrated courses in mathematics and other subjects. In M. J. Zweng, T. Green, J. Kilpatrick, H. O. Pollak, & M. Suydam (Eds.), *Proceedings of the Fourth International Congress on Mathematical Education* (pp. 247–249). Boston, MA: Birkhäuser.

Oberle, M. (2016). Politikwissenschaft als Bezugsdisziplin der Politischen Bildung. In M. Oberle & G. Weißeno (Eds.), *Politische Bildung. Politikwissenschaft und Politikdidaktik: Theorie und Empirie* (pp. 17–29). Wiesbaden: Springer Fachmedien Wiesbaden.

Ojose, B. (2011). Mathematics Literacy: Are We Able To Put The Mathematics We Learn Into Everyday Use? *Journal of Mathematics Education*, *4*(1), 89–100.

O'Neill, J. (2002). The Rhetoric of Deliberation: Some Problems in Kantian Theories of Deliberative Democracy. *Res Publica*, *8*, 249–268.

Print, M. (2013). Competencies for Democratic Citizenship in Europe. In M. Print & D. Lange (Eds.), *Civic and Political Education: Vol. 3. Civic education and competences for engaging citizens in democracies* (pp. 37–51). Rotterdam: Sense Publishers.

Reinhardt, S. (2013). Teaching for Democratic Learning. In M. Print & D. Lange (Eds.), *Civic And Political Education: Vol. 3. Civic education and competences for engaging citizens in democracies* (pp. 99–109). Rotterdam: Sense Publishers.

Reinhardt, S. (2018). *Politik Didaktik: Handbuch für die Sekundarstufe I und II. Fachdidaktik*. Berlin: Cornelsen.

Strømskag, H. (2021). Prospective teachers solving a percentage problem: An analysis of the construction of a praxeology. In B. Barquero, I. Florensa, P. Nicolás, & N. Ruiz-Munzón (Eds.), *Extended abstracts spring 2019: Advances in the Anthropological Theory of the Didactic.* Basel: Birkhäuser.

Zweng, M. J., Green, T., Kilpatrick, J., Pollak, H. O., & Suydam, M. (Eds.). (1983). *Proceedings of the Fourth International Congress on Mathematical Education*. Birkhäuser Boston. https://doi.org/10.1007/978-1-4684-8223-2

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